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SENSITIVITY OF LOW-ENERGY PION-NUCLEON SCATTERING TO A PION-PION RESONANCE

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Recently attention has been called to the possibility of explaining nucleon electromagnetic structure quantitatively by the introduction of a pion-pion resonance in the $T=1$, $J=1$ state.¹ Such a resonance should also affect pion-nucleon scattering. General expressions relating pion-nucleon to π - π scattering within the framework of double dispersion relations have been developed,^{2,3} and some detailed applications have been made to low-energy π - N scattering.^{4,5} In particular, it has been found that the $P_{3/2, 3/2}$ π - N resonance is insensitive to π - π scattering.^{4,5} However, the applications made to date have not been entirely satisfactory because the short-range π - N interaction is not well understood.

In this Letter we wish to point out that in π - N scattering the $J=1/2$, P states are extremely sensitive to the π - π resonance. The π - π contribution to these states near threshold is greater than the contribution to the $P_{T=3/2, J=3/2}$ state,

roughly in the ratio⁶

$$P_{1/2, 1/2} : P_{3/2, 1/2} : P_{1/2, 3/2} : P_{3/2, 3/2} \sim 9 : -4.5 : -2 : 1.$$

We further wish to argue that the π - π resonance parameters which fit the nucleon electromagnetic structure make it difficult to obtain agreement with experiment in several π - N scattering states. This conclusion is especially strong in the $P_{1/2, 1/2}$ state where, in addition to being large, the π - π contribution represents an attractive, long-range interaction which should lead to a large positive phase shift no matter what form the unknown short-range interaction may take.

Let us consider the $J=1/2$, P -state amplitude,

$$f_{J=1/2}^P(W) \equiv (q/\mu)^{-3} e^{i\delta} \sin \delta, \quad (1)$$

where W and q are the total energy and the pion momentum in the center-of-mass system. In Table I we list the contribution to f at threshold

Table I. Comparison of Chew-Low theory, and Frazer-Fulco π - π resonance contribution, with π - N scattering experiments for the $J=1/2$, P amplitudes f [Eq. (1)] at threshold. The experimental numbers are from the 1958 CERN conference.^a

	Chew-Low	π - π	Total	Exp.
$f_{1/2}^P(T=1/2)$	-0.14	+0.41	+0.27	-0.038 \pm 0.038
$f_{1/2}^P(T=3/2)$	-0.035	-0.21	-0.245	-0.044 \pm 0.005

^a1958 Annual International Conference on High-Energy Physics at CERN, edited by B. Ferretti (CERN Scientific Information Service, Geneva, 1958).

from the well-known Chew-Low theory, and the additional contribution we obtain from π - π resonance scattering in the $N + \bar{N} \rightarrow 2\pi$ channel as calculated within the framework of double dispersion relations. Since the π - π contribution is unfamiliar and crucial to our argument, it is necessary to give some details concerning it. By the procedures of reference 5 it can be expressed as

$$\pi\pi_{f_{J=1/2}}^P(M+\mu)\left(\frac{T=1/2}{T=3/2}\right) = \binom{-2}{1} \frac{\mu^3}{8\pi t_R(M+\mu)} \times \left[2\gamma_1 \left(6 + \frac{3\mu}{M} + \frac{8M\mu}{t_R} \right) + \gamma_2 \left(16 + \frac{24\mu}{M} + \frac{3t_R}{M^2} \right) \right], \quad (2)$$

where M and μ are the nucleon and pion masses, t_R is the energy of the π - π resonance, and the γ_i are related to the nucleon electromagnetic spectral functions¹ g_i^V and the π - π form factor¹ F_π by

$$\gamma_1 = -8\pi \left(\frac{g_1^V(t_R)}{e|F_\pi(t_R)|^2} \right) \left(1 - \frac{4\mu^2}{t_R} \right)^{-3/2}, \quad (3)$$

$$\gamma_2 = -8\pi \left(\frac{Mg_2^V(t_R)}{e|F_\pi(t_R)|^2} \right) \left(1 - \frac{4\mu^2}{t_R} \right)^{-3/2} \quad (4)$$

We shall use the π - π resonance width and position ($t_R = 11.5\mu^2$) with which Frazer and Fulco obtained the best results for nucleon electromagnetic structure. According to their work¹ the π - π resonance essentially contributes a factor $|F_\pi|^2$ to the "magnetic" spectral function g_2^V , and we can estimate $g_2^V/|F_\pi|^2$ in (4) quite accurately from the value which it takes in the absence of π - π scattering.^{7,8} We estimate the "charge" term $g_1^V/|F_\pi|^2$ in the same way, although the estimate is less reliable in this case. With this procedure, 2/3 of the numerical value of Eq. (2) is given by the "magnetic" contribution so that our results are not especially sensitive to the uncertainty in g_1^V . We find that $\gamma_1 = -6.9$ and $\gamma_2 = -15.6$, which leads to the π - π results in Table I.

The π - π terms we have obtained at threshold appear to be much too large, and we proceed to strengthen this conclusion by some further consideration of the $P_{1/2,1/2}$ amplitude. When the quantities in Table I are calculated at energies above threshold, we find that the $P_{1/2,1/2}$ amplitude passes through resonance at less than 200-Mev

kinetic energy, and that at this energy the attractive π - π contribution is at least 5 times the repulsive Chew-Low contribution. A closer analysis, of the type carried out in reference 5, can be made. Such analysis shows that the contributions we have considered in this Letter are dominated by the nearest singularities in the complex energy plane, which correspond to the longest range interactions in the problem, and that the terms we have not taken into account are of shorter range. Crude estimates of the short-range effects indicate they are attractive, but the main point is that even if other factors lead to a net "repulsive core," the attraction which we have found at longer ranges should still produce a large net positive phase shift. Thus the Frazer-Fulco resonance leads to a low-energy $P_{1/2,1/2}$ amplitude which is in disagreement with experiment.

We place somewhat less weight on the apparent disagreement in the $P_{3/2,1/2}$ state (Table I), because an attraction in the unknown short-range region could conceivably bring about agreement in this case.

Frazer and Fulco¹ found that the nucleon electromagnetic structure does not determine the π - π resonance parameters very precisely, so we must consider variations of the resonance width and energy. The quantities γ_1 and γ_2 are nearly constant over the range of parameters which are consistent with the electromagnetic data. It then follows from Eq. (2) that the π - π contribution varies $\sim t_R^{-1}$. The "range" of the π - π contribution also decreases $\sim t_R^{-1}$. Thus, although the discrepancy in the $P_{1/2,1/2}$ state persists at larger t_R , it becomes smaller and the argument concerning the "range of interaction" becomes weaker. It is also possible that the π - π channel is more complicated than has been imagined; for example, the existence of a 3π bound state⁹ might force a re-examination of all the previous work on π - π effects.

Turning now to the recent work of Bowcock et al.⁴ (B), we wish to argue that although these authors obtained a π - π contribution about 8 times smaller than that in Table I, their work actually strengthens our conclusion. There are two main factors which account for their smaller π - π term:

(i) The authors of B worked at $t_R = 22.4\mu^2$, which is double our value and halves the π - π contribution as explained above.

(ii) The remaining factor of ~ 4 arises because the π - π resonance width was determined by different criteria in B than in Frazer and Fulco. In our work, following Frazer and Fulco,¹ the

π - π resonance width is determined by the anomalous magnetic moment. In B, no attempt is made to fit the magnetic moment. Instead, the resonance width is fit to the π - N S -wave energy dependence (the S -wave thresholds are taken from experiment, and then the more long-range-dependent curvature above threshold is calculated). This procedure leads to a much narrower resonance than the Frazer-Fulco treatment would give at $t_R = 22.4\mu^2$, and if the anomalous moment were calculated at $t_R = 22.4\mu^2$ with this narrow resonance it would come out much too large. When the fit to the energy dependence of the nucleon form factors is made, the narrow resonance width is responsible for the above-mentioned factor of 4 reduction of the $N + \bar{N} \rightarrow 2\pi$ amplitude in B.

Thus the moderate π - π contributions in B are obtained at the expense of failing to explain the nucleon magnetic moment; if the π - π resonance were fit to the magnetic moment the work in B indicates that the π - N S state as well as $J=1/2$, P state would disagree with experiment.

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SINGLE PION PRODUCTION IN 0.96-Bev π^- - p INTERACTIONS*

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Photographs of interactions in a hydrogen bubble chamber (field 13.5 kilogauss) irradiated by 0.96-Bev negative pions from the Cosmotron were kindly supplied by Professor Steinberger. We have measured and analyzed 415 events of the neutron reaction

$$\pi^- + p \rightarrow n + \pi^+ + \pi^-,$$

and 267 events of the proton reaction

$$\pi^- + p \rightarrow p + \pi^0 + \pi^-.$$

Experimental values quoted in this Letter all refer to the barycentric system.

Alles-Borelli, Bergia, Ferreira, and Waloscek¹ have shown, from measurements of other photo-

graphs in the same irradiation, evidence for the presence of a resonant state $T=J=3/2$; effects of this state have been calculated by Lindenbaum and Sternheimer.²

If this were the only resonant state involved, fast negative pions ("fast" designating a momentum ≥ 325 Mev/c) would originate mostly as recoils from $(n\pi^+)_{3/2}$ or $(p\pi^0)_{3/2}$; they would, accordingly, have the same angular distribution in both reactions, and (from Clebsch-Gordan coefficients) would be twice as numerous in the proton as in the neutron reaction. Figure 1 shows that the angular distributions are quite different, and the observed ratio of proton to neutron events with fast π^- is 0.9 ± 0.2 . It follows then, that at